where $Bi_m = \alpha_m R/\lambda_m$ is the Biot number; α_m is the heat transfer coefficient between the last (m-th) layer of the heat exchanger and the medium that washes over it; T_{ext} is the temperature of the medium external to the heat exchanger.

Coordinate functions for the successive approximations are obtained from Eq. (26).

In this case, solutions of the form (15) and (20) are to be taken in the following forms:

$$\overline{T}^*_{1i}(\rho,\ s,\ p) = T_{ext} - b_1(s,\ p)\varphi_{1i}(\rho)\ (i = \overline{1,\ m}), \tag{28}$$

$$T_{ni}(\rho,\ z) = T_{ext} - \sum_{k=1}^{n} f_k(z)\varphi_{ki}(\rho)\quad (i = \overline{1,\ m}). \tag{29}$$

The expressions (28) and (29), with coordinate functions obtained from equations (26) and (27), satisfy the boundary conditions and all the junction conditions. The unknown coefficients $b_1(s, p)$ and $f_k(z)$ are to be determined in such a way that the initial differential equations are satisfied in an optimum manner. For this purpose one can use the Bubnov–Galerkin orthogonal method (for determining $b_1(s, p)$) and the Kantorovich method (for determining $f_k(z)$ $(k = \overline{1, n})$.

It should be pointed out in conclusion that the approach outlined here makes it possible to solve effectively heat exchange junction problems for boundary conditions varying in time and with respect to z, as well as for fluid temperatures at the channel entrances varying with respect to ρ and with respect to the time, and for initial heat-carrier temperatures dependent on the coordinates ρ and z.

NOTATION

T, temperature; T_{I1}, T_{I2}, T_I, initial temperatures; T_w, outer wall temperature; T_{ext}, temperature of external medium; W_{av}, average velocity; x, r, longitudinal and transverse coordinates; τ, time; r_1, $r_2 = R$, distances to inner and outer walls; $\rho = r/R$, dimensionless coordinate; a, smaller of the diffusivity coefficients a_1 and a_2; $Pe = 6RW_{av}/a$, Peclet number; $z = (1/Pe)x/R$, dimensionless coordinate; $Fo = a\tau/R^2$, Fourier number; λ, coefficient of thermal conductivity; m, number of heat exchanger layers; H(η), Heaviside function; η, argument of Heaviside function; α, heat transfer coefficient; $Bi = \alpha R/\lambda$, Biot number.

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THE DYNAMICS OF THE FREEZING OVER OF UNDERGROUND PIPES

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The article suggests a method of calculating the unsteady process of freezing over of an underground pipe transporting a freezing liquid.

Pipeline transport of water, aqueous solutions and suspensions under conditions of low ambient temperatures may be accompanied by their freezing. The formation of an ice layer

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on the inner pipe surface causes increased hydraulic resistance. In view of that the prediction of freezing over of pipes under extreme climatic and technological conditions has to be included in the pipeline project.

A considerable number of authors dealt with the heat exchange between a liquid and a wall whose temperature is lower than the freezing point of the liquid. Articles [1, 2] dealt with the nonsteady heat exchange between a liquid and a cold flat wall taking the growing ice layer into account. The wall temperature was assumed to be constant. In [3-5] an analogous problem was solved for flow in a pipe, in [6] a similar problem was considered for the case of a constant heat flux into the soil. The effect of the evolution of the temperature field in the environment for the plane and the axisymmetric problem was taken into account in [7, 8]. In [9] the method of [8] was developed with the phase transitions in the surrounding soil taken into account. The present article constitutes a further development of the method of [9] in connection with laminar and turbulent flow in pipes, and it substantiates some assumptions that had been used in [9].

We deal with the heat exchange of a liquid flowing through an underground pipe with the surrounding soil. At the instant of starting the soil is frozen and has a temperature $T_f < T_p$. The temperature of the liquid at the inlet is $T_0 > T_p$. In the region $0 \leq z < b$ the pipe is not frozen over and the equation of the influx of heat is written in the form [9]

$$C_1^j \frac{\partial \theta}{\partial t} + C_2^j \frac{\partial \theta}{\partial z} = q^j + C_3^j; \quad \begin{matrix} 0 \leqslant z < b; \\ j = 1,\ 2; \end{matrix} \tag{1}$$

$$\theta|_{z=0} = 1. \tag{2}$$

Here,

$$\theta = \frac{T - T_p}{T_0 - T_p}; \quad \theta_j = \frac{T_j - T_p}{T_0 - T_p}, \quad t = \frac{\tau \varkappa_2}{R_0^2};$$

$$C_1^j = \frac{\rho c \varkappa_2}{2\lambda_j}\left(\frac{R_i}{R_0}\right)^2; \quad C_2^j = \frac{\rho c G}{2\pi\lambda_j}; \quad C_3^j = \frac{\rho g G i}{2\pi\lambda_j (T_0 - T_p)};$$

$$q^j = \frac{1}{2\pi}\int_0^{2\pi} \left.\frac{\partial \theta_j}{\partial r}\right|_{r=1} d\varphi; \quad r = \frac{\bar{r}}{R_0};$$

$$i = \begin{cases} \dfrac{0{,}31 G^2}{4\pi^2 g a^5 (\lg \mathrm{Re} - 1)^2}, & \mathrm{Re} > \mathrm{Re}_{cr}; \\ \dfrac{16 G^2}{\pi^2 g a^5 \mathrm{Re}}, & \mathrm{Re} < \mathrm{Re}_{cr}. \end{cases}$$

In the region $b < z < L$ on the inner pipe wall, an ice layer forms, and the radius of its inner surface is $\delta(z, \tau)$. The equation of heat influx for the liquid in this zone is

$$C_1^j x \frac{\partial \theta}{\partial t} + C_2^j \frac{\partial \theta}{\partial z} = -\alpha\theta + C_3^j; \tag{3}$$

$$\theta|_{z=b} = \theta_B. \tag{4}$$

Here $\alpha = \delta\tilde{\alpha}/\lambda_j$; $x = (\delta/R_i)^2$.

The heat transfer coefficient from the liquid to the inner pipe surface $\tilde{\alpha}$ is determined by the following relations:

$$\tilde{\alpha} = \begin{cases} 0{,}023 \dfrac{\lambda_\ell}{2a} \mathrm{Re}^{0,8} \mathrm{Pr}^{0,33}, & \mathrm{Re} > \mathrm{Re}_{cr}; \\ 1{,}62 \dfrac{\lambda_\ell}{2a}\left(\dfrac{4G_w}{\pi}\right)^{0,33}, & \mathrm{Re} < \mathrm{Re}_{cr}, \quad G_w > 10; \\ 3{,}78 \dfrac{\lambda_\ell}{2a}, & \mathrm{Re} < \mathrm{Re}_{cr}, \quad G_w \leqslant 10; \end{cases}$$

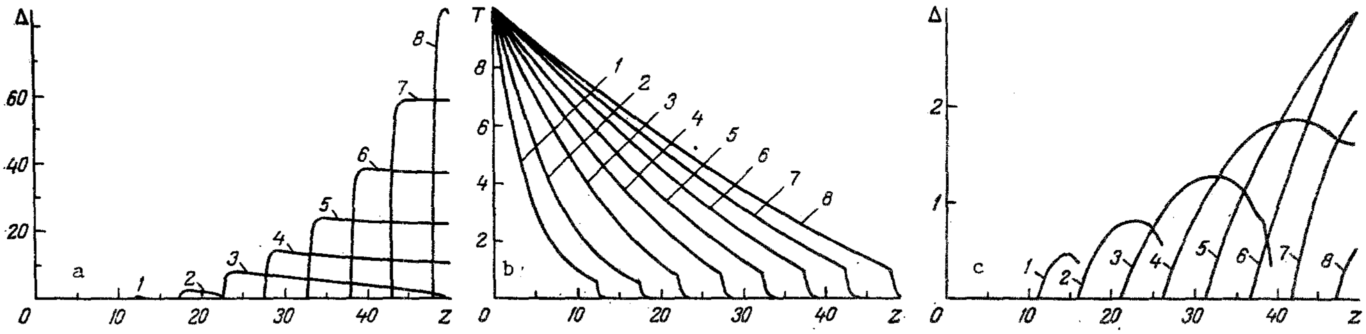

Fig. 1. Dependences of the thickness of the ice layer $\Delta = R_i - \delta$ (mm) (a, c) and of the temperature of the liquid T (°C) (b) on the longitudinal coordinate z (km) for different instants: a, b) variant I: 1) τ = 6.51 h; 2) 11.61; 3) 25.18; 4) 50.3; 5) 92.08; 6) 156.9; 7) 252.4; 8) 383 h; c) variant II: 1) τ = 8.21 h; 2) 13.39; 3) 19.6; 4) 26.00; 5) 35.95; 6) 46.5; 7) 59.3; 8) 74 h.

$G_w = \rho c G/\lambda_\ell z$; a is the radius of the inner cross section of the pipe. The statement of the problem of heat conduction for the surrounding soil corresponds to [9] and is not presented here.

The growth of the ice layer on the inner pipe surface is determined by Stefan's condition:

$$l\rho_{ic}\frac{\partial\delta}{\partial\tau} = \tilde{\alpha}(T - T_p) + \lambda_{ic}\frac{\partial T_{ic}}{\partial r}\bigg|_{r=\delta}; \tag{5}$$

$$\delta|_{\tau=\tau_0(z)} = R_i. \tag{6}$$

Here, $\tau_0(z)$ is the time of arrival of the front of filling the pipe in the section z.

Let us first regard the solution of the equation of heat influx for the liquid (1), (2). The region $0 \le z < b$ can be divided into two zones. In the first zone $0 \le z < b'$ melted soil lies next to the pipe. The dimensionless temperature of the liquid at the boundary of this zone is determined after [9] by the relation

$$\theta(b') = \exp[-F(b')]\left\{\frac{C_3^1}{C_2^1}\int_0^{b'}\exp(F(z))\,dz\right\}. \tag{7}$$

Here,

$$F(z) = \begin{cases} \dfrac{2}{C_2^1\sqrt{D_e}}\left(\operatorname{arctg}\dfrac{2a_2z + a_1}{\sqrt{D_e}} - \operatorname{arctg}\dfrac{a_1}{\sqrt{D_e}}\right), & D_e > 0; \\ \dfrac{1}{C_2^1\sqrt{-D_e}}\ln\left|\dfrac{(2a_2z + a_1 - \sqrt{-D_e})(a_1 + \sqrt{-D_e})}{(2a_2z + a_1 + \sqrt{-D_e})(a_1 - \sqrt{-D_e})}\right|, & D_e < 0; \end{cases}$$

$$D_e = 4a_2a_0 - a_1^2; \quad a_0 = \ln s_0 + \frac{1}{\alpha_1}; \quad a_1 = u/s_0;$$

$$a_2 = -\left(\ln s_0 + \frac{ub'}{s_0}\right)/b'^2; \quad u \equiv \frac{\partial s}{\partial z}\bigg|_{z=0};$$

$$\alpha_1 = \left[\lambda_1\left(\frac{1}{\tilde{\alpha}R_i} + \frac{1}{\lambda_{in}}\ln\frac{R_0}{R_e}\right)\right]^{-1}.$$

The system of differential equations for determining s_0, u, b' was given in [9].

The dimensionless temperature of the liquid on the frontal boundary of the ice layer z = b is determined by the following expression:

$$\theta_d \equiv \theta(b) = \theta_f + C_3^2Q(t') + [\theta(b') - \theta_f - C_3^2Q(t')]\exp\left(\frac{b' - b}{C_2^2Q(t')}\right), \tag{8}$$

where

$$t' = t - t_0;\ t_0 = \frac{\tau_0\varkappa_2}{R_0^2};\quad Q(t') = \frac{\lambda_2}{\tilde{\alpha}R_i} + \frac{\lambda_2}{\lambda_{in}}\ln\frac{R_0}{R_e} + \frac{1}{q_w(t')}.$$

Here and henceforth $\tilde{\alpha}$ is averaged over the interval of integration. If in the interval of integration a change of the hydraulic conditions occurs, then integration is carried out separately for zones with unchanging conditions.

The function q_w characterizing nonsteady heat exchange with the surrounding single-phase soil is determined in accordance with [9]. It is obvious that if the temperature of the phase transition of the capillary liquid of the soil and the temperature of the liquid transported through the pipe coincide and the heat insulation of the pipe is thin, $b' = b$ and the temperature on the frontal boundary of the ice layer is determined by expression (7).

Let us now turn to the solution of Eq. (5). Going over to dimensionless variables and using an approximate solution for the temperature gradient on the inner boundary of the ice layer, we obtain

$$A_1 \frac{\partial x}{\partial t'} = \frac{\tilde{\alpha}}{\tilde{\alpha}_0} \theta \sqrt{x} + \frac{A_2 \theta_f}{C - C_1 \ln x + \frac{1}{q_w(t')}}. \tag{9}$$

Here,

$$A_1 = \frac{l \rho_{ic} \varkappa_2 R_i}{2\tilde{\alpha}_0 R_0^2 (T_0 - T_p)}; \; A_2 = \frac{\lambda_2}{R_i \tilde{\alpha}_0};$$

$$C = \frac{\lambda_2}{\lambda_{in}} \ln \frac{R_0}{R_e}; \; C_1 = \frac{\lambda_2}{2\lambda_{ic}}; \; \tilde{\alpha}_0 = \tilde{\alpha}(R_i); \; x|_{t'=0} = 1.$$

Let us consider the solution of (9) for small t' which corresponds to the profile of the ice layer near the front of filling. With a view to the expression for the radius of thermal influence in this case [11] we have: $R(t') \simeq 1 + \sqrt{6t'}$. We write the function q_w in the following manner: $q_w = \sqrt{\frac{2}{3t'}}$. In addition we bear in mind that with $t' \to 0$ $x \to 1$, $\tilde{\alpha} \to \tilde{\alpha}_0$, the ice layer grows much more slowly than the thickness of the heated soil layer. In view of that Eq. (9) assumes the form

$$A_1 \frac{\partial x}{\partial t'} = \theta + \frac{A_2 \theta_f}{C + \sqrt{\frac{3t'}{2}}}. \tag{10}$$

When t' is small, the expression for the dimensionless temperature in the zone that is free of freezing over is:

$$\theta = \theta_f + C_3^2 q_c + (1 - \theta_f - C_3^2 q_c) \exp\left(-\frac{z}{C_3^2 q_c}\right). \tag{11}$$

Here, $q_c = \sqrt{\frac{3}{2} t'} + C + A_2.$

Let us consider the evolution of the dimensionless temperature in the section z during the period when an ice layer exists in this section. At the instant that the ice layer appears, which corresponds to the instant of arrival of the front of filling in this section, its dimensionless temperature is

$$\theta_{on} = C_3^{ic} \left[-C_3^{ic} + \theta_f + C_3^2 (C + A_2) + (1 - \theta_f - C_3^2 \times \right.$$
$$\left. \times (C + A_2)) \exp\left(-\frac{s_0^0}{C_2^2 (C + A_2)}\right)\right] \exp\left(\frac{s_0^0 - z}{C_2^{ic}}\right). \tag{12}$$

Here,

$$C_2^{ic} = \frac{\rho c G}{2\pi R_i \tilde{\alpha}_0}; \; C_3^{ic} = \frac{\rho g G i}{2\pi R_i \tilde{\alpha}_0 (T_0 - T_p)};$$

s_0^0 is the coordinate of the front of filling in which the formation of the ice layer begins.

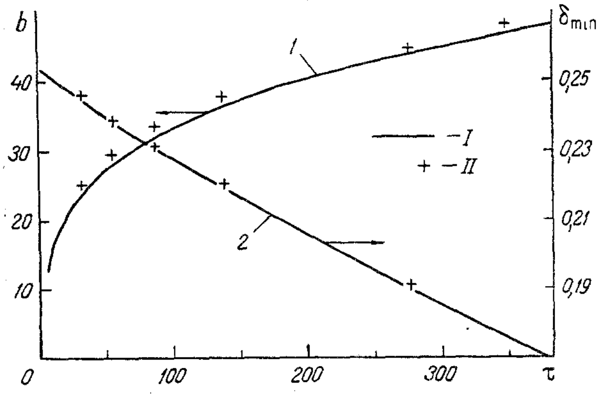

Fig. 2. Dependences of the coordinate of the front edge of the ice b (km) and of the minimal radius of the inner surface of the ice layer δ_{min} (m) on time: 1) b(τ); 2) $\delta_{min}(\tau)$; I) complete method; II) simplified method.

The condition of the onset of growth of the layer is obtained from (9):

$$\theta + \frac{A_2\theta_f}{C} = 0. \tag{13}$$

When we use (11), it follows from the above-said that

$$s_0^0 = C_2^2 (C + A_2) \ln \left| \frac{\theta_f + C_3^2 (C + A_2) - 1}{\theta_f + C_3^2 (C + A_2) + A_2\theta_f/C} \right|. \tag{14}$$

When we substitute this expression into (12), we obtain

$$\theta_{on} = C_3^{ic} - (C_3^{ic} + A_2\theta_f/C) \exp\left(\frac{s_0^0 - z}{C_2^{ic}}\right). \tag{15}$$

At the instant that the ice layer disappears in the section z, the temperature of the flow in it is

$$\theta_{in} = \theta_f + C_3^2 q_{cin} + (1 - \theta_f - C_3^2 q_{cin}) \exp\left(-\frac{z}{C_2^2 q_{cin}}\right). \tag{16}$$

Here, $q_{cin} = \sqrt{\frac{3}{2} t'_{in}} + C + A_2$; $t'_{in} = t_{in} - t_0$.

Assuming that the temperature of the flow in the section z changes linearly with time, we represent Eq. (10) in the form

$$A_1 \frac{\partial x}{\partial t'} = \theta_{on} + (\theta_{in} - \theta_{on}) \frac{t'}{t'_{in}} + \frac{A_2\theta_f}{C + \sqrt{\frac{3}{2} t'}}. \tag{17}$$

When we integrate this equation, we obtain the following expression:

$$x = 1 + \frac{\varkappa_2}{A_1 V R_0^2} \left[\theta_{on} \xi + (\theta_{in} - \theta_{on}) \frac{\xi^2}{2\xi_{in}} + \frac{4A_2\theta_f}{3} \frac{V R_0^2}{\varkappa_2} \left(\sqrt{\frac{3}{2} \frac{\xi \varkappa_2}{V R_0^2}} - C \ln \left| 1 + \frac{1}{C} \sqrt{\frac{3}{2} \frac{\xi \varkappa_2}{V R_0^2}} \right| \right) \right]. \tag{18}$$

Here $\xi = VR_0^2/\kappa_2\, t'$ is the distance between the given section and the front of filling; $\xi_{in} = VR_0^2\, t'_{in}/\kappa_2$ is the same for the instant that the layer disappears. Obviously, when $\xi = \xi_{in}$, x = 1. When we substitute these values into (18), we obtain a transcendental equation for determining ξ_{in}:

$$\xi_{in}(\theta_{on} + \theta_{in}) = \frac{8A_2\theta_f}{3} \frac{V R_0^2}{\varkappa_2} \left(C \ln \left| 1 + \frac{1}{C} \sqrt{\frac{3}{2} \frac{\xi_{in} \varkappa_2}{V R_0^2}} \right| - \sqrt{\frac{3}{2} \frac{\xi_{in} \varkappa_2}{V R_0^2}} \right) \tag{19}$$

Expression (18) describes the profile of the ice layer at the initial period of ice formation, and also near the front of filling.

When the value of t'_{in} obtained by Eq. (19) is larger than t_a (the limit of applicability of the solution of (18)), we have to put

$$t'_{in} = t_a \cdot \ \theta_{in} = C_3^{ic} + [-C_3^{ic} + \theta_f + C_3^2 q_{ca} + (1 - \theta_f - C_3^2 q_{ca}) \times$$
$$\times \exp\left(-\frac{b}{C_2^2 q_{ca}}\right)\Big] \exp\left(\frac{b-z}{C_2^{ic}}\right);$$
$$q_{ca} = \sqrt{\frac{3}{2} t_a} + C + A_2.$$

The system of equations (1)-(4), (9), (10) fully describes the temperature and ice conditions of the pipe. For its integration a program in FORTRAN-V was worked out. The solutions for small t' were obtained with the aid of relations (11), (15)-(18). This eliminates the computing difficulties arising at the initial instant of ice formation. First, with the aid of (14), the coordinate of the place of origin of the ice layer is determined. Then, for the initial time interval $[0, t_a]$ we seek the profiles of the ice layer from relations (18), (19). By numerical integration of the problem (3), (4) we determine for the instants $t > t_a$ the temperature field of the liquid above the ice layer at the given instant, and with the aid of (9) we determine the increase of the ice layer; then the procedure is repeated.

Figure 1 shows the results of the calculations by the mentioned program. The calculations were carried out for the following initial data: $Q = 400$ m³/h; $R_0 = 0.266$ m; $R_e = 0.2635$ m; $R_i = 0.2535$ m; $L = 50$ km; $c = 4.18$ kJ/(kg·°K); $\lambda_\ell = 0.559$ W/(m·°K); $\lambda_{ic} = 2.32$ W/(m·°K); $\lambda_{in} = 2.09$ W/(m·°K); $\kappa_2 = 0.585 \cdot 10^{-2}$ m²/h; $\omega = 300$ kg/m³; $T_f = 4$°C; $T_0 = +10$°C; $T_p = 0$°C; $\ell = 335$ kJ/kg; $\lambda_{in} = 209$ W/(m·°K); $t_a = 0.025$.

Kinematic viscosity ν and density ρ of the liquid were adopted for variant I equal to the corresponding characteristics of water ($\nu = 0.647 \cdot 10^{-2}$ m²/h; $\rho = 1000$ kg/m³). For variant II we calculated the case of a suspension based on water ($\nu = 1.5$ m²/h; $\rho = 1400$ kg/m³). Variant I is characterized by turbulent flow ($Re \approx 1.5 \cdot 10^4$), II is characterized by laminar flow ($Re \approx 700$). It can be seen from the presented graphs that a characteristic feature of variant I is the abrupt increase of the thickness of the ice layer in the region of its front edge; further downstream along the pipe its thickness changes only slightly (Fig. 1a). An inherent trait of the temperature of the liquid is the abrupt drop in the region of the front edge of the ice (Fig. 1b). These effects are due to the fact that the heat transfer coefficient from the liquid to the inner pipe wall is larger when the flow is turbulent. Variant II (laminar flow) is characterized by a small longitudinal temperature gradient of the liquid in the region of the front edge of the ice and by correspondingly even growth of its thickness (Fig. 1c).

The above-mentioned features of freezing over in turbulent flow make it possible to simplify the method of calculating it. Since in the region of freezing over, except in a narrow zone near the front edge, the temperature of the liquid remains practically unchanged, it can be determined from Eq. (3)

$$\theta = C_3^i/\alpha; \ b \leqslant z \leqslant L. \tag{20}$$

In view of this, the equation of the growth of the ice layer (9) assumes the form

$$A_1 \frac{\partial x}{\partial t'} = \frac{C_3^i \sqrt{x}}{\alpha_0} + \frac{A_2 \theta_f}{C - C_1 \ln x + 1/q_w(t')}. \tag{21}$$

Here, $\alpha_0 = \tilde{\alpha}_0 \delta / \lambda_j$.

Obviously, integrating the given equation is a simpler problem than integrating the system (3), (9). The regularity of motion of the front edge of the ice layer can be found on the basis of the following considerations. Melting of the front edge occurs on account of the lowering of the temperature of the flow from Θ_d to θ_e, i.e., the temperature at which the heat fluxes on the inner surface of the ice layer are in equilibrium:

$$\theta_e = \frac{\alpha_0}{\alpha} \frac{A_2 \theta_f}{\sqrt{x}(C - C_1 \ln x + 1/q_w)}. \tag{22}$$

Hence, when we construct the relation of thermal balance, we obtain

$$\frac{db}{dt} = \left(\frac{R_0}{R_i}\right)^2 \frac{G}{\pi \varkappa_2 \left[1 + \frac{l \rho_{ic}}{c\rho (T_0 - T_p)} \frac{1-x}{(\theta_d - \theta_e)}\right]}. \tag{23}$$

Figure 2 shows a comparison of the results of determining the coordinate of the front edge of the ice layer and of its maximal thickness obtained with the aid of the simplified method (21)-(23) and the more complete method described above. It can be seen from the presented graphs that the simplified method yields practically the same results as the more complete one. Yet for its realization much less computer time and memory capacity are required.

NOTATION

T, temperature of the liquid; T_p, temperature of the phase transition of the liquid; ρ, density of the liquid; λ_ℓ, thermal conductivity of the liquid; c, specific heat capacity of the liquid; T_0, temperature of the liquid at the inlet to the pipe; G, volumetric flow rate; i, hydraulic gradient; E, mechanical equivalent of heat; R_i, inner pipe radius; R_e, outer pipe radius; R_0, outer radius of the heat insulation; z, longitudinal coordinate; $\overline{r}$, radial coordinate; T_j, temperature of the j-th zone of soil; λ_j, thermal conductivity of the j-th zone of soil; κ_j, thermal diffusivity of the j-th zone of soil ($j = 1$: melted zone, $j = 2$: frozen zone); Re, Reynolds number; Re_{cr}, Reynolds number at which laminar flow changes into turbulent flow; $\tilde{\alpha}$, heat transfer coefficient from the liquid to the inner pipe wall; b, longitudinal coordinate of the front edge of the ice layer; δ, radius of the inner surface of the ice layer; ℓ, specific heat of the phase transition ice—water; ρ_{ic}, weight of the ice in unit volume of frozen liquid; T_{ic}, temperature of the frozen liquid; λ_{ic}, thermal conductivity of the frozen liquid; λ_{in}, thermal conductivity of the insulation; R, dimensionless radius of thermal influence; g, acceleration of gravity; s, dimensionless radius of melting of the soil around the pipe; s_0, the same for the initial section of the pipeline; T_f, natural soil temperature; τ, time; Pr, Prandtl number.

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